

Syllabus
Math 01.354 - Introduction to Topology

CATALOG DESCRIPTION:

Math 01.354 Introduction to Topology, 3 s.h.

Prerequisites: Math 01.330 (Introduction to Real Analysis I) with a grade of C- or better

This course includes the properties of general topological spaces, metric spaces, separation, compactness, connectedness and the Heine-Borel and Bolzano-Weierstrass theorems.

OBJECTIVES:

This one-semester three-credit course in Introductory Topology will have three general interconnected objectives. First, as it has become increasingly apparent that topology is one of the major branches of modern mathematics, this course will provide a firm foundation in topology to enable the student to continue more advanced study in this area. Second, as several important areas of mathematics, in particular modern analysis, depend upon or are clarified by certain topics in topology, this course will present and emphasize those topics in order to aid the student in his future mathematical studies. Finally, this course hopes to expose the students to both mathematical rigor and abstraction, giving them an opportunity further to develop mathematical maturity.

CONTENT:

- 1. The Real Number Line**
 - 1.1 Axioms Defining Real Numbers
 - 1.2 Least Upper Bound
 - 1.3 Cantor's Theorem
 - 1.4 Convergent Sequences
 - 1.5 Bolzano-Weierstrass Theorem

- 2. Continuous Functions**
 - 2.1 Continuity and Uniform Continuity
 - 2.2 Continuity on the Plane
 - 2.3 Hilbert Curve
 - 2.4 Brouwer's Fixed-Point Theorem
 - 2.5 Homeomorphism

- 3. Topology of the Line and Plane**
 - 3.1 Accumulation (or Adherent) Points
 - 3.2 Boundary of a Set
 - 3.3 Open and Closed Sets
 - 3.4 Interior and Closure
 - 3.5 Dense and Meager Sets
 - 3.6 Completeness
 - 3.7 Compactness
 - 3.8 Heine-Borel Theorem

4. Topological Spaces

- 4.1 Open Sets and Topologies
- 4.2 Accumulation (or Adherent) Points
- 4.3 Closure and Neighborhood
- 4.4 Convergent Nets and Directed Sets
- 4.5 Bases and Subbases for a Topology
- 4.6 Coarser and Finer Topologies
- 4.7 Subspaces and Relative Topologies
- 4.8 Continuity in Terms of Open Sets

5. Metric Spaces

- 5.1 Distances and Metrics
- 5.2 Diameters and Open Spheres
- 5.3 Equivalent Metrics
- 5.4 Euclidean m -space

6. Compactness

- 6.1 Covers and Compact Sets
- 6.2 Finite Intersection Property and Compactness
- 6.3 Sequential Compact Sets
- 6.4 Compactness in Metric Spaces

7. Separation Axioms

- 7.1 T_1 -Spaces
- 7.2 Hausdorff Spaces
- 7.3 Regular Spaces
- 7.4 Normal Spaces

8. Connectedness

- 8.1 Connectedness on \mathbb{R}
- 8.2 Applications
- 8.3 Separated sets

TEXTS:

1. Buskes, Gerard & Arnold Van Rooij, TOPOLOGICAL SPACES FROM DISTANCE TO NEIGHBORHOOD, Springer-Verlag, New York, NY, 1997.
2. Andre L. Yandl & Adam Bowers, ELEMENTARY POINT-SET TOPOLOGY: A TRANSITION TO ADVANCED MATHEMATICS (Aurora: Dover Modern Math Originals), Dover Publications, 2016.
3. John B. Conway, A COURSE IN POINT SET TOPOLOGY (Undergraduate Texts in Mathematics), Springer, 2014.

Updated: 5.1.18

Reviewed: 04/2021