

Syllabus

**Math 01.330 - Introduction to Real Analysis I**

**CATALOG DESCRIPTION:**

**Math 01.330 Introduction to Real Analysis I, 3 s.h.**

Prerequisites: Math 01.230 (Calculus III) and Math 01.300 (Mathematical Proof Writing) with a grade of C- or better in both courses

This course introduces students to rigorous mathematical thought processes in Analysis, and prepares them for more advanced courses in the related areas. Topics included are: sets, functions, the real number system, sequences, limits, continuity and derivatives.

**OBJECTIVES:**

Students will demonstrate the ability to use rigorous mathematical thought processes in the following areas: sets, functions, sequences, limits, continuity, and derivatives.

**CONTENTS:**

**1.0 Introduction**

- 1.1 Real numbers
  - 1.1.1 Absolute values, triangle inequality
  - 1.1.2 Archimedean property, rational numbers are dense
- 1.2 Sets and functions
  - 1.2.1 Set relations, Cartesian product
  - 1.2.2 One-to-one, onto, and inverse functions
- 1.3 Cardinality
  - 1.3.1 One-to-one correspondence
  - 1.3.2 Countable and uncountable sets
- 1.4 Methods of proof
  - 1.4.1 Direct proof
  - 1.4.2 Contrapositive proof
  - 1.4.3 Proof by contradiction
  - 1.4.4 Mathematical induction

**2.0 Sequences**

- 2.1 Convergence
  - 2.1.1 Cauchy's epsilon definition of convergence
  - 2.1.2 Uniqueness of limits
  - 2.1.3 Divergence to infinity

- 2.1.4 Convergent sequences are bounded
- 2.2 Limit theorems
  - 2.2.1 Summation/product of sequences
  - 2.2.2 Squeeze theorem
  - 2.2.3 Cauchy sequences and Cauchy criteria for convergence
  - 2.2.4 Completeness axiom
  - 2.2.5 Bounded monotone sequences are convergent
- 2.3 Subsequences and limit points
  - 2.3.1 Bolzano-Weierstrass theorem
- 2.4 Supremum and infimum
  - 2.4.1 A bounded set has a unique least upper bound

### **3.0 Continuity**

- 3.1 Limits of functions
  - 3.1.1 Definition of continuity based on sequences
  - 3.1.2 Definition of continuity based on open intervals
  - 3.1.3 Summation/product/composition of continuous functions
- 3.2 Properties of continuous functions on a closed interval
  - 3.2.1 A continuous function is bounded
  - 3.2.2 A continuous function attains its supremum/infimum
  - 3.2.3 Intermediate-value theorem
  - 3.2.4 Uniform continuity

### **4.0 Differentiation**

- 4.1 Derivatives
  - 4.1.1 Limit definition of a derivative
  - 4.1.2 Rules of differentiation
  - 4.1.3 Chain rule
  - 4.1.4 Higher-order derivatives
- 4.2 Properties of differentiable functions
  - 4.2.1 Differentiability implies continuity
  - 4.2.2 Continuously differentiable functions

#### **TEXT:**

\*Michael Reed, FUNDAMENTAL IDEAS OF ANALYSIS, WILEY & SON, 1998

Bartle, Robert G. & Sherbert, Donald R., INTRODUCTION TO REAL ANALYSIS, third ed., John Wiley & Sons, Inc., 2000.

Mattuck, Arthur, INTRODUCTION TO ANALYSIS, first ed., Prentice Hall, 1999.