

Syllabus
Math 01.503 - Number Theory

CATALOG DESCRIPTION:

Math 01.503 Number Theory 3 S.H.

This course includes prime numbers, linear and polynomial congruences, law of quadratic reciprocity, algebraic numbers and integers, other topics in number theory and unsolved problems in number theory. This course may not be offered annually.

OBJECTIVES:

It is the purpose of this course to present to the student an introduction to an area of pure mathematics which, although it does not abound in practical application, has intrigued many non-professionals people, as well as the greatest mathematicians from the time of the ancient Greeks to the present.

CONTENT:

1. Basic Concepts

- 1.1 Properties of the integers
- 1.2 Divisibility: definition and the division algorithm
- 1.3 Greatest common divisor
- 1.4 Least common multiple
- 1.5 The Euclidean algorithm
- 1.6 Primes
- 1.7 The fundamental theorem of arithmetic

2. Linear Diophantine Equations

- 2.1 Solution of $ax + by = c$

3. Congruences

- 3.1 Definition
- 3.2 Arithmetic properties
- 3.3 The linear congruences $ax = b \pmod{m}$
- 3.4 Residue classes
- 3.5 Systems of linear congruences and the Chinese remainder theorem
- 3.6 Euler's Phi function
- 3.7 Introduction to higher order congruences
- 3.8 Applications:
 - 3.8.1 Tests for divisibility useful in arithmetic
 - 3.8.2 Checks for the basic operations of arithmetic

4. Euler's Theorem

- 4.1 Complete systems of residues
- 4.2 Reduced systems of residues
- 4.3 Euler's and Fermat's theorems
- 4.4 The exponent to which "a" belongs \pmod{m}

5. Perfect numbers

- 5.1 The sigma and tau functions
- 5.2 Even perfect numbers

6. Nonlinear Diophantine Equations

6.1The Pythagorean problems

6.2Fermat's last theorem

7.Continued Fractions

7.1Simple continued fractions, finite and infinite

7.2The closeness of approximations by convergents

7.3The Pell equation

8.Applications

8.1Cryptology

8.2The calendar problem

TEXTS:

The following are indicative of texts suitable for the course:

Burton, David, ELEMENTARY NUMBER THEORY, W. C. Brown Publishing Company, Dubuque, IA, 1989.

Niven, I. and Zuckerman, H., THEORY OF NUMBERS, John Wiley Publishing Company, NYC, 1990.

Rosen, Kenneth, ELEMENTARY NUMBER THEORY, Addison-Wesley Publishing Company, Reading, MA, 1988.